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DEVELOPMENT OF A METHOD FOR NEUTRONICS TREATMENT OF NON-MULTIPLYING ZONES IN PEBBLE BED REACTORS

Abderrafi M. Ougouag/Idaho National
Laboratory, Idaho Falls, ID 83415, USA

Farzad Rahnema/Georgia Institute of
Technology, Atlanta, GA 30332, USA

Hikaru Hiruta/Idaho National
Laboratory, Idaho Falls, ID
83415, USA

Dingkang Zhang/Georgia
Institute of Technology,
Atlanta, GA 30332, USA

Charles A. Wemple/Idaho
National Laboratory, Idaho
Falls, ID 83415, USA

David W. Nigg/Idaho National
Laboratory, Idaho Falls, ID 83415, USA

ABSTRACT

In this paper, a diffusion-transport hybrid nodal method is presented that can effectively treat non-multiplying zones in pebble bed reactors. The new method seamlessly combines the analytic coarse-mesh finite difference (CMFD) diffusion formulation and a transport theory-based response matrix formulation while retaining the properties and structure of the CMFD diffusion solver. The resulting combined formulation is utilized in selected non-multiplying nodes in order to avoid using poorly homogenized data. Numerical results demonstrate the accuracy of the new methodology.

INTRODUCTION

The preparation of diffusion theory data for regions within a reactor core or reflector that contain no multiplying media requires special care. Indeed, since no source is present therein, the regions must be treated as “driven” ones that receive all their neutrons from outside. The most common approach to preparing homogenized data for non-multiplying regions (e.g., reflectors and baffles) were reviewed by Smith [1]. One of these techniques is the extended assembly calculation method [1] in which a two-zone approach is used that includes at least one heterogeneous fuel assembly and the reflector (and baffle if applicable). That approach provides acceptable accuracy for use in nodal computations. However it requires the pre-computation of the homogenized data and is not compatible with the requirement of online generation of diffusion data, as needed in the PEBBED code methodology. The foregoing discussion pertains mainly to ordinary non-multiplying zones such as reflectors. However, when localized strong absorbers

are present, such as control rods or burnable poisons, the spatially-dependent spectral effects are even more severe and more care must be taken in the preparation of the diffusion data. In the case of the pebble bed reactor, the treatment of the latter situation has been approached in two ways. In one approach, the Method of Equivalent Cross Sections [2,3], the control rod and a region around it are isolated and both a transport and a diffusion computation are conducted adjusting the diffusion data to produce a match for the transport results. In the second approach, response functions are obtained from auxiliary computations, usually a set of detailed transport theory solutions for the region, then the “response functions” thus derived are used to provide internal boundary conditions for the whole core diffusion computations [4,5]. Both of these approaches are difficult to implement and require significant intervention from the analyst.

This paper presents a mathematically rigorous method that builds on the response matrix approach while retaining the properties and the structure of the diffusion theory-based neutronics solver within the PEBBED [6] code. The new method dispenses with the concept of internal boundary conditions and can (and is) implemented in a way that is fully transparent to the user. The new method combines into a single framework the diffusion theory solution and the local transport solution. Because of the single framework, the method retains the ease of use and the computational efficiency of the original diffusion solution method.

NOMENCLATURE

Areas and volumes

Δx_i = Size of i^{th} coarse-mesh node

Ω_i = i^{th} coarse-mesh node

Macroscopic cross sections

- $\Sigma_R^g(x)$ = Group g removal cross section
- $\Sigma_s^{g' \rightarrow g}(x)$ = Scattering cross section (group g' to g)
- $\Sigma_f^g(x)$ = Fission cross section
- ν_f^g = Number of fission neutrons per fission event
- $\Sigma_{R,i}^g$ = Homogenized removal cross section in i^{th} node

Fluxes and sources

- Φ_i^g = i^{th} coarse node average scalar flux (energy group g)
- ϕ_i^{g,s_i} = Homogeneous surface flux on surface s_i
- $\phi_{h,i}^{g,s_i}$ = Heterogeneous surface flux on surface s_i
- J_i^{g,s_i} = Net current on surface s_i
- J_i^{out,g,s_i} = Partial outgoing current from surface s_i
- J_i^{in,g,s_i} = Partial incoming current from surface s_i
- \bar{S}_i^g = Node average source term
- $S_{i,l}^g$ = the l^{th} spatial moment of the source term
- $\tilde{S}_{\text{eff},i}^{g,R}$ = effective source term for the quantity $Z_i^{g,X}$

Functions and factors

- $G_i^{g,\pm\pm}$ = Green's function
 - $(G_{\pm,i}^g)_l$ = l^{th} spatial moment of Green's function
 - $R_{g' \rightarrow g, s_i' \rightarrow X}^{Z_i^{g,X}}$ = Partial current response function for the quantity $Z_i^{g,X}$.
 - f_i^{g,s_i} = Interface flux discontinuity factor on surface s_i
- ### Coefficients
- $D_{i,s_i}^g, E_{i,s_i}^g, H_{i,s_i}^g$ = Coefficients for CMFD diffusion interface current formulation
 - $\alpha_i^{g,s_i}, \beta_i^{g,s_i}, \gamma_i^g$ = Coefficients for RF transport interface current formulation
 - $\tilde{\alpha}_i^{g,s_i}, \tilde{\beta}_i^{g,s_i}, \tilde{\gamma}_i^g, \tilde{E}_{i,s_i}^g, \tilde{H}_{i,s_i}^g$ = Coefficients for hybrid interace current formulation
 - Θ_i^{g,s_i} = Coefficient for RFNB equation.

METHODOLOGY

Direct CMFD Method from Analytical Diffusion Solution

The Direct CMFD method was first introduced (in general simplified notations) by Chao [7]. Here we present its explicit derivation in notations that match the neutronics application in order to prepare the stage for the corresponding developments in transport theory.

Let us consider the k -eigenvalue problem for the few-group neutron balance equation:

$$\frac{dJ^g}{dx} + \Sigma_R^g(x)\phi^g(x) = S^g(x), \quad 0 \leq x \leq a, \quad (1)$$

$$J^g(0) = 0, \quad J^g(a) = C^g \phi^g(a), \quad (2)$$

where

$$S^g(x) = \sum_{g' \neq g} \Sigma_s^{g' \rightarrow g}(x)\phi^{g'}(x) + \frac{\chi^g}{k_{\text{eff}}} \sum_{g'=1} \nu_f^{g'} \Sigma_f^{g'} \phi^{g'}(x), \quad (3)$$

and C^g is a problem- and methodology-dependent constant (i.e. 0.5 for diffusion theory). Integrating Eq.(1) over a given coarse-mesh node $\{\Omega_i : x_i \leq x \leq x_{i+1}\}$ results in the discretized nodal balance equation:

$$J_i^{g,R} - J_i^{g,L} + \Sigma_{R,i}^g \Delta x_i \Phi_i^g = \Delta x_i \bar{S}_i^g, \quad i = 1, \dots, N_x, \quad (4)$$

where Δx_i is the size of the coarse node Ω_i . In order to obtain a system of equations for the coarse-mesh node-averaged scalar fluxes Φ_i^g (i spanning the domain), we derive expressions for J_i^g as a function of the Φ_i^g in adjacent nodes by means of the Analytic Nodal Green's Function method [8] or its ILLICO equivalent [9]. On the left surface of the i^{th} coarse-mesh node, the homogeneous surface flux $\phi_i^{g,L}$ can be expressed in terms of the Green's function for the diffusion operator evaluated at node boundaries, $G_i^{g,\pm\pm}$ (see ANNEX A) and its double moments within the node (for sources):

$$\phi_i^{g,L} = G_i^{g:-} J_i^{g,L} - G_i^{g:+} J_i^{g,R} + \sum_{l=1}^L (G_{-,i}^g)_l S_{i,l}^g. \quad (5)$$

Similarly, on the right surface of the $i-1^{\text{th}}$ coarse-mesh, we have:

$$\phi_{i-1}^{g,R} = G_{i-1}^{g:+} J_{i-1}^{g,L} - G_{i-1}^{g:-} J_{i-1}^{g,R} + \sum_{l=1}^L (G_{+,i-1}^g)_l S_{i-1,l}^g. \quad (6)$$

We use the relationship of continuity (or discontinuity) of homogeneous scalar fluxes on this shared surface by introducing interface flux discontinuity factors f_i^g [1]:

$$f_{i-1}^{g,R} \phi_{i-1}^{g,R} = f_i^{g,L} \phi_i^{g,L}. \quad (7)$$

Utilizing the relationship given by Eqs.(4) and (7), we derive the desired expression for $J_i^{g,L}$:

$$J_i^{g,L} = -D_{i,L}^g \Phi_i^g + D_{i-1,R}^g \Phi_{i-1}^g + E_{i,L}^g \bar{S}_i^g - E_{i-1,R}^g \bar{S}_{i-1}^g - H_{i,L}^g \sum_{l=1}^L (G_{-,i}^g)_l S_{i,l}^g + H_{i-1,R}^g \sum_{l=1}^L (G_{+,i-1}^g)_l S_{i-1,l}^g. \quad (8)$$

By substituting Eq.(8) and the similar expression for $J_i^{g,R}$ into Eq.(4), and repeating for all nodes, there results a tridiagonal system of equations for the coarse-mesh node-averaged scalar fluxes, Φ_i^g .

Direct CMFD Method from Transport Based Response Functions

The integral quantities $Z_i^{g,X}$ in energy group g , averaged over the volume or area of interest (denoted by X , which stands for either node volume Δx_i for the node of index i or for its surface s_i), can be formulated by means of the response functions (RFs) according to [4,5]

$$Z_i^{g,X} = \sum_{g'} \sum_{s'_i} R_{g' \rightarrow g, s'_i \rightarrow X}^{Z_i^{g',X}} J_i^{in, g', s'_i}, \quad (9)$$

where J_i^{in, g, s_i} is the partial incoming current from the surface s_i into node i and $R_{g' \rightarrow g, s'_i \rightarrow X}^{Z_i^{g',X}}$ represents the partial current RF for the given integral property $Z_i^{g',X}$. Note that in this application these integral quantities are the node averaged scalar fluxes, Φ_i^g , the heterogeneous surface fluxes, $\phi_{h,i}^{g, s_i}$, and the partial outgoing currents, J_i^{out, g, s_i} . The above formulation needs to be reformulated into a within-group formulation in order to allow its seamless incorporation within the 1D CMFD diffusion formulation. For instance, the quantities averaged over the right surface are written as:

$$Z_i^{g,R} = R_{g \rightarrow g, R \rightarrow R}^{Z_i^{g,R}} J_i^{in, g, R} + R_{g \rightarrow g, L \rightarrow R}^{Z_i^{g,R}} J_i^{in, g, L} + \tilde{S}_{eff,i}^{Z_i^{g,R}}, \quad (10)$$

where $\tilde{S}_{eff,i}^{Z_i^{g,R}}$ is the effective source containing the partial incoming currents for the other energy groups, which must be updated at every iteration. After all the relationships for the quantities $Z_i^{g,X}$ are formulated, the nodal balance equation can be derived based on the response function (RFNB equation). It is given by:

$$\Theta_i^{g,R} J_i^{g,R} - \Theta_i^{g,L} J_i^{g,L} + \Phi_i^g = \Theta_i^{g,R} \tilde{S}_{eff,i}^{J_i^{out, g, R}} + \Theta_i^{g,L} \tilde{S}_{eff,i}^{J_i^{out, g, L}} + \tilde{S}_{eff,i}^{\Phi_i^g}, \quad (11)$$

where $\Theta_i^{g,R}$ and $\Theta_i^{g,L}$ are coefficients calculated by means of the RFs. To find formulations for the interface currents, J_i^g , as a function of Φ_i^g , we impose the continuity of heterogeneous scalar fluxes at a given interface:

$$\phi_{h,i-1}^{g,R} = \phi_{h,i}^{g,L}, \quad (12)$$

where

$$\phi_{h,i-1}^{g,R} = R_{g \rightarrow g, L \rightarrow R}^{\phi_{h,i-1}^{g,R}} J_{i-1}^{in, g, L} + R_{g \rightarrow g, R \rightarrow R}^{\phi_{h,i-1}^{g,R}} J_{i-1}^{in, g, R} + \tilde{S}_{eff,i-1}^{\phi_{h,i-1}^{g,R}}, \quad (13)$$

$$\phi_{h,i}^{g,L} = R_{g \rightarrow g, L \rightarrow L}^{\phi_{h,i}^{g,L}} J_i^{in, g, L} + R_{g \rightarrow g, R \rightarrow L}^{\phi_{h,i}^{g,L}} J_i^{in, g, R} + \tilde{S}_{eff,i}^{\phi_{h,i}^{g,L}}. \quad (14)$$

The desired expression for $J_i^{g,L}$ can be derived by utilizing Eqs. (10)-(14):

$$J_i^{g,L} = \beta_{i-1}^{g,R} \Phi_{i-1}^g - \beta_i^{g,L} \Phi_i^g - \beta_{i-1}^{g,R} \tilde{S}_{eff,i-1}^{\Phi_{i-1}^g} + \beta_i^{g,L} \tilde{S}_{eff,i}^{\Phi_i^g} + \alpha_{i-1}^{g,R} \tilde{S}_{eff,i-1}^{J_{i-1}^{out, g, R}} - \alpha_i^{g,L} \tilde{S}_{eff,i}^{J_i^{out, g, L}} - \gamma_i^g \tilde{S}_{eff,i-1}^{\phi_{h,i-1}^{g,R}} + \gamma_i^g \tilde{S}_{eff,i}^{\phi_{h,i}^{g,L}}. \quad (15)$$

The substitution of Eq.(15), and of a similar expression for $J_i^{g,R}$, into Eq.(11) results in a tridiagonal system of equations for Φ_i^g . This similarity in structure of the two direct CMFD methods gives the opportunity to couple the two methodologies transparently. The only remarkable difference between the two methods is that there is no homogenized parameter involved in the RF method. The next section discusses the additional condition that must be met in order to seamlessly incorporate transport theory based response matrix method into CMFD analytic diffusion method.

Hybrid Nodal Interface Current Formulation

In the previous sections, the interface currents have been expressed in a straightforward manner as functions of node averaged scalar fluxes when the interface under consideration is located between the same types of nodes. For instance, the current at the interface between two diffusion nodes can be formulated by following the CMFD analytic nodal Green's function technique, whereas that at the interface between two transport nodes can be derived based on the RF formulation, as just shown. However, if the target interface is located between a diffusion node and a transport node, the derivation of the expression for the current must take into account a mix of diffusion and transport effects. The derivation steps of such an expression are consistent with those of the previously derived formulations that utilize the continuity of surface averaged scalar fluxes. However, it is necessary to use two different formulations for surface averaged scalar fluxes: on one side of the interface the scalar flux is based on the CMFD diffusion formulation whereas on the other side the scalar flux is based on the RF formulation. For instance, on the interface between a CMFD diffusion node on the left and a transport node on the right, one must enforce the continuity of scalar flux by introducing the discontinuity factor only on the CMFD diffusion side:

$$f_{i-1}^{g,R} \phi_{i-1}^{g,R} = \phi_{h,i}^{g,L}. \quad (16)$$

By utilizing Eqs.(4), (6), (10), (11), (14), and (16), one can derive a hybrid expression for the interface current:

$$J_i^{g,L} = \tilde{\beta}_{i-1}^{g,R} \Phi_{i-1}^g - \tilde{\beta}_i^{g,L} \Phi_i^g - \tilde{E}_{i-1,R}^g \bar{S}_i^g + \tilde{\beta}_i^{g,L} \tilde{S}_{eff,i}^{\Phi_i^g} + \tilde{\alpha}_i^{g,L} \tilde{S}_{eff,i}^{J_i^{out, g, L}} - \tilde{\gamma}_i^g \tilde{S}_{eff,i}^{\phi_{h,i}^{g,L}} + \tilde{H}_{i-1,R}^g \sum_{l=1}^{L_i} (G_{+i-1}^l)_l S_{i-1,l}^g. \quad (17)$$

By taking into account all the possible diffusion-transport hybrid configurations, a tri-diagonal system of equations for

the node averaged scalar fluxes is obtained utilizing these interface current formulations (Eqs.(8), (15), and (17)), the CMFD nodal balance equation (Eq.(4)), and the RMNB equation (Eq.(11)).

NUMERICAL RESULTS

To demonstrate the performance of the new hybrid nodal method, as implemented in the INL code PEBBED [6], we present numerical results for a 1D model problem inspired by the design of the PBMR. The 1D PBMR model core, shown in Figure 1, consists of an inner reflector (100cm), a fuel region (85cm), and an outer reflector (50cm). We define two uniform coarse nodes in the inner reflector, five in the fuel region, and two in the outer reflector. The innermost coarse node in the outer reflector (Node 8) contains the control element having a thickness of 1.5cm, shown in Figure 2. The reflective and vacuum ($J^n = 0$) boundary conditions are applied on the inner and outer boundaries, respectively. The nuclide concentrations of this test problem are shown in Table 1.

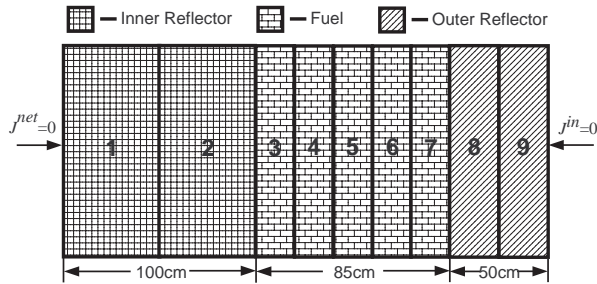


Figure 1. 1D PBMR Model Geometry.

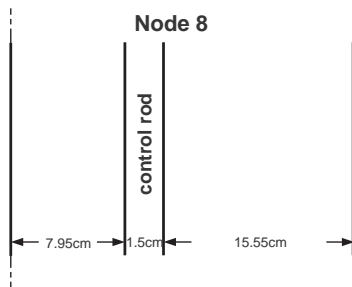


Figure 2. Geometric Configuration of Node 8.

The few group homogenized cross sections and response functions are generated in two different ways by means of MCNP [10] or modified MCNP transport computations.

The first computation is a full domain transport computation (i.e., a full core solution). The data generated from this case (diffusion theory parameters, discontinuity factors and response functions) are fully compatible with the transport solution and are *considered reference data* in this work.

Table 1. Nuclide Concentrations of 1D PBMR Model

	Nuclide	Density ($\times 10^{24}$ atoms/cm ³)
Inner reflector	C	8.97470×10^{-2}
	U-234	1.16811×10^{-7}
Fuel	U-235	1.19359×10^{-5}
	U-238	1.10859×10^{-4}
	O	2.45830×10^{-4}
	Si	2.75487×10^{-4}
	C	5.23349×10^{-2}
Outer reflector	C	9.02480×10^{-2}
	B	3.20000×10^{-6}
Control rod	C	7.24720×10^{-2}

In the second approach the diffusion theory data and the discontinuity factors are generated using the conventional approach in which a full core computation is not performed, but rather data are prepared for limited regions using the conventional approach of assuming reflective boundary conditions. The response functions are generated using a local transport solution for unit incoming partial currents at each surface (using energy and angular unfolding based on saved spectra and angular distribution from either the reference solution or a two node solution). For the non-multiplying nodes, a driving boundary condition is used for the generation of the diffusion theory data and the discontinuity factors. All the data generated in this model problem assume a thermal energy cut-off at 2.38 eV.

The full core MCNP solution mentioned above is taken as the reference solution in this work.

Four additional solutions can be obtained from the data generated as described above. These three solutions are:

(1) a “Conventional-data CMFD diffusion” solution, using conventional CMFD with diffusion theory treatment in all nodes and with data generated according to the traditional approach of limited zones with reflective or driving boundary conditions, as appropriate,

(2) a “Hybrid conventional-data CMFD” solution with diffusion CMFD treatment in all nodes except for transport treatment through RF-based CMFD in the controlled node, using data generated per the conventional method of limited zones with reflective boundaries for the diffusion nodes and using RFs derived for isolated nodes for the transport node; this case does not make use of discontinuity factors and its data do not conform to modern equivalence theory,

(3) a “Hybrid exact-data CMFD” solution with diffusion CMFD treatment in all nodes, except for transport treatment through RF-based CMFD in the controlled node; this case uses “exact” reference data generated from the MCNP reference solution for the diffusion nodes and RFs derived from the reference

MCNP solution for the controlled node; this case also uses discontinuity factors and updates them through the rehomogenization approach, and

(4) a “diffusion CMFD Reference” or “CMFD with exact data” solution that uses diffusion-based CMFD in all nodes with “exact” diffusion theory data generated from the reference MCNP solution results and based on the application of modern equivalence theory augmented with the rehomogenization step for the discontinuity factors.

The controlled node referred to in the cases descriptions above is the node labeled “8” in Figure 1.

Based on Generalized Equivalence Theory [1], the nodal calculations for the “diffusion CMFD Reference” case is expected to reproduce the MCNP-equivalent solution provided that the discontinuity factors are also exact. In order to find accurate discontinuity factors, the rehomogenization technique [11] was applied. This technique involves an additional outer iteration loop for updating the discontinuity factors. This process usually converges in 10-20 iterations. Note that the rehomogenization is used only for the discontinuity factors since all other few group data items are exact. The use of exact group data effectively simulates the rehomogenization for those data.

Figures 3 and 4 show the node-averaged group scalar fluxes obtained using the new hybrid CMFD method. For comparison, these figures include the MCNP solution as well as the results from all four cases defined above. In these figures (as well as Figs. 5 and 6), the various cases are labeled “Conventional CMFD Diffusion” for Case (1), “Hybrid w/o Exact Data for Case (2), “Hybrid” for Case (3), and “CMFD Diffusion w/Exact Data” for Case (4).

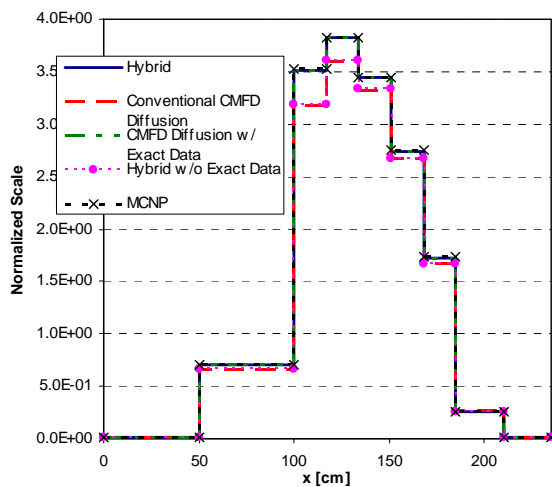


Figure 3. Fast Group Node Average Scalar Flux

The quantitative difference between the new method and MCNP solutions can be observed in Figures 5 and 6. As seen in these figures, it is apparent that the new hybrid method,

when used with data that conform to the prescriptions of modern equivalence theory, is highly accurate. It nearly matches the solution quality of the CMFD diffusion with exact data and rehomogenization. The maximum relative difference occurs within the outer reflector region in both energy groups; however, these maximum errors are less than 1% and occur in regions where no fissions occur. The relative differences within the fuel region are very small, always less than 0.2%. In contrast, the solution by the conventional CMFD diffusion calculation has a maximum relative difference of over 10% in both energy groups. These results demonstrate not only the superiority of the new hybrid CMFD method over the current methodology but also the effectiveness of the rehomogenization method for few group data.

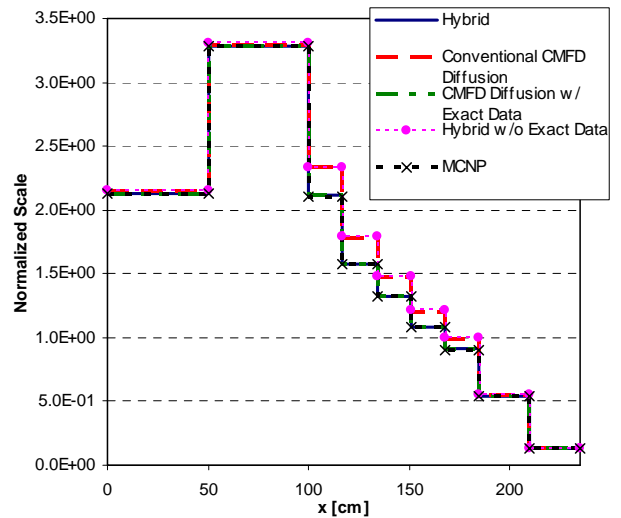


Figure 4. Thermal Group Node Average Scalar Flux

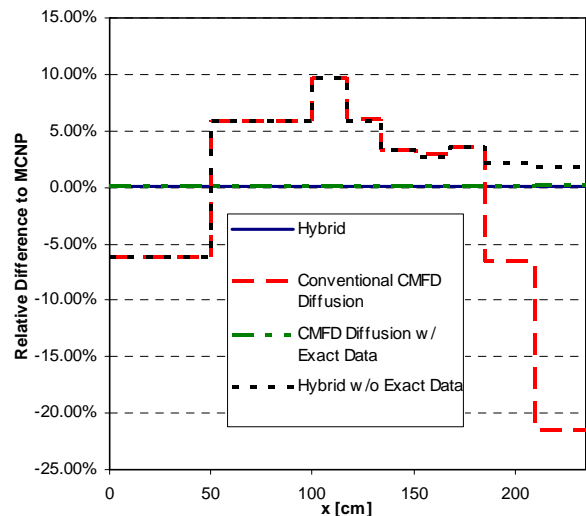


Figure 5. Fast Group Flux: Relative Difference to MCNP

Further observation of Figures 5 and 6 reveals that the use of the RF-based CMFD method in the controlled node, instead of diffusion theory-based CMFD, improves the solutions in that node. Another important observation is the

obvious fact that the proper preparation of diffusion theory data is paramount in the production of correct diffusion theory solutions, as previously observed by the authors in the context of the generation of data for the pebble bed reactor. In particular, it must be noted that the use of point spectrum codes without spatial/spectral effects correction seems inadequate for the task and that modern equivalence theory methods must be adapted and applied.

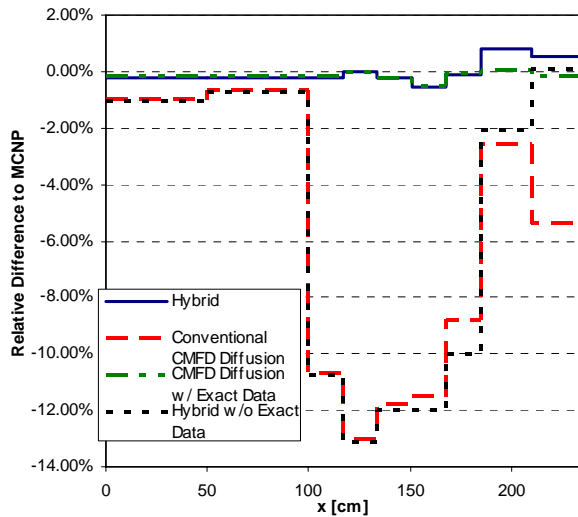


Figure 6. Thermal Group Scalar Flux: Relative Difference to MCNP Solution

Table 2 shows the eigenvalue obtained by each method. As seen in this table, the eigenvalue obtained by the new hybrid method, as well as by the CMFD with exact and rehomogenized data, is very close to that obtained by MCNP, having 0.2% of relative difference.

Table 2. Eigenvalue Obtained by Each Method.

	k_{eff}
Hybrid	1.17154
Conventional CMFD	1.15566
CMFD w/ exact data	1.17137
Hybrid w/o exact data	1.15625
MCNP	1.16934

The agreement between the new method and the reference values shown in Table 2 appears impressive. However, it is important to analyze whether the relative differences while being quantitatively small are qualitatively

small enough to provide confidence in criticality predictions. For this purpose, the MCNP code was used to obtain the eigenvalue while modeling node 8 without a control rod in it. In this latter situation, the value obtained for k_{eff} is 1.28111. Next, the relative change in multiplication factor is computed (i.e., the reactivity, $\Delta k / k$ is obtained) that is due to the control rod insertion and withdrawal. A quantitative comparison of relative eigenvalue changes is shown in Table 3. In this table, one can observe that the relative error in the eigenvalue obtained using the new hybrid nodal method is much smaller than the eigenvalue change due to the presence/absence of the control rod.

Table 3. Relative Changes in Eigenvalue $\Delta k / k$

Method-based relative error in k_{eff}	0.2%
$\Delta k/k$ due to CR insertion	-9.6%
$\Delta k/k$ due to CR withdrawal	8.7%

DISCUSSION

A new hybrid CMFD method has been developed that can effectively treat non-multiplying zones. The method has been demonstrated with a 1D model problem representative of pebble bed reactors. Two different direct CMFD methods, the analytic nodal diffusion Green's function technique and the transport based RF method, are seamlessly connected with each other retaining the properties and the structure of the CMFD diffusion solver in the PEBBED code. This is achieved using a mathematically rigorous formulation. The numerical results obtained with the sample problem show the high accuracy of the new hybrid CMFD method. It is demonstrated that the new method enjoys almost the same level of accuracy as that of the analytic CMFD diffusion method with exact and rehomogenized data. Because of this result the method is currently undergoing extension to 3D cylindrical geometry for implementation into the PEBBED code as a neutronics solver option.

It is noteworthy that the new hybrid nodal method does not require the homogenization of the control rod nodes. Instead, it requires the pre-computation of response functions. The latter are material-, geometry- and context-dependent.

It must be born in mind that the new method, as others before it, is ultimately only as good as the data that are fed into it. One of the important conclusions of this paper is that while the new method can improve the solution in the nodes in which RF-based CMFD is used, the overall diffusion theory solution requires the preparation of diffusion data based on modern equivalence theory if reliably and

consistently good results are to be achieved. The methods currently used for pebble bed reactor design and analysis do not yet apply the modern theory, and they should be brought up to date in this regard.

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REFERENCES

- [1] Smith, K. S., "Assembly Homogenization Techniques for Light Water Reactor Analysis," *Progress in Nuclear Energy*, **17**, pp. 303-335 (1986).
- [2] Scherer, W. and Neef, H. J., "Determination of Equivalent Cross Sections for Representation of Control Rod Regions in Diffusion Calculations," Jül 1311, (1976).
- [3] Fen, V., Lebedev, M., Sarytchev, V., and Scherer, W., "Modeling of Neutron Absorbers in High Temperature Reactors by Combined Transport-Diffusion Methods," Jül 2573, (1992).
- [4] Mosher, S. W., Rahnama, F., and Forget, B., "Monte Carlo Adaptation of a Heterogeneous Coarse Mesh Transport Method," *Trans. Am. Nucl. Soc.*, **89**, 310, (2003) [see also erratum, **90**, 601, (2004)].
- [5] Forget, B., Rahnama, F., and Mosher, S. W., "A Heterogeneous Coarse Mesh Solution for the 2-D NEA

C5G7 MOX Benchmark Problem," *Progress in Nuclear Energy*, **45**, No. 2-4, 233-254, (2004).

- [6] Terry, W. K., Gougar, H. D., and Ougouag, A. M., "Direct Deterministic Method for Neutronics Analysis and Computation of Asymptotic Burnup Distribution in a Recirculating Pebble-Bed Reactor," *Ann. Nucl. Energy*, **29**, pp. 1345-1364 (2002).
- [7] Chao, Y. A., "A Theoretical Analysis of the Coarse Mesh Finite Difference Representation in Advanced Nodal Methods," Proc. Int. Conf. Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications (M&C'99), Madrid, Spain, Sept. 1999, Vol. 1, p.117 (1999).
- [8] Ougouag, A. M., "A Coarse-Mesh Nodal Method for Multigroup Multidimensional Neutron Diffusion Computations," M.S. Thesis, University of Illinois, 1981.
- [9] Rajic, H. L., and A. M. Ougouag, "ILLICO: A Nodal Neutron Diffusion Method for Modern Computer Architectures," *Nuclear Science and Engineering*, **103**, 4, 392-408 (1989).
- [10] X-5 MONTE CARLO TEAM, "MCNP-A General Purpose Monte Carlo N-Particle Transport Code, Version 5," LA-UR-03-1987, Los Alamos National Laboratory (2003).
- [11] Rahnama, F. and Nichita, E. M., "Leakage Corrected Spatial (Assembly) Homogenization Technique," *Ann. Nucl. Energy*, **24**, No. 6, pp. 477-488 (1997).

ANNEX A

GREEN'S FUNCTION FOR ANALYTIC DIFFUSION SOLUTION

The Green's function used in the analytic CMFD diffusion method is the solution of the following Neumann problem [8]:

$$\frac{d^2 G^g(u, u_0)}{du^2} - \frac{\Sigma_R^g}{D^g} G^g(u, u_0) = -\frac{1}{D^g} \delta(u - u_0), \quad (A1)$$

$$\left. \frac{dG^g}{du} \right|_{u=-a} = 0, \quad \left. \frac{dG^g}{du} \right|_{u=+a} = 0, \quad -a \leq u \leq a, \quad (A2)$$

where D^g is the diffusion coefficient. The solution of this problem is given by:

$$G^g(u, u_0) = \begin{cases} \frac{1}{\kappa^g D^g} \frac{\cosh[\kappa^g(a - u_0)] \cosh[\kappa^g(a + u)]}{\sinh[2\kappa^g a]}; & u \leq u_0 \\ \frac{1}{\kappa^g D^g} \frac{\cosh[\kappa^g(a + u_0)] \cosh[\kappa^g(a - u)]}{\sinh[2\kappa^g a]}; & u \geq u_0 \end{cases} \quad (A3)$$

where

$$\kappa^g = \sqrt{\frac{\Sigma_R^g}{D^g}}. \quad (A4)$$

Based on this solution, the explicit forms of the discretized Green's function coefficients in Eqs. (5) and (6) can be written as:

$$G_i^{g:-} = \frac{1}{\kappa_i^g D_i^g} \frac{\cosh[\kappa_i^g \Delta x_i]}{\sinh[\kappa_i^g \Delta x_i]}, \quad (A5)$$

$$G_i^{g:+} = \frac{1}{\kappa_i^g D_i^g} \frac{1}{\sinh[\kappa_i^g \Delta x_i]}, \quad (A6)$$

$$G_{i-1}^{g:+} = \frac{1}{\kappa_{i-1}^g D_{i-1}^g} \frac{1}{\sinh[\kappa_{i-1}^g \Delta x_{i-1}]}, \quad (A7)$$

$$G_{i-1}^{g:++} = \frac{1}{\kappa_{i-1}^g D_{i-1}^g} \frac{\cosh[\kappa_{i-1}^g \Delta x_{i-1}]}{\sinh[\kappa_{i-1}^g \Delta x_{i-1}]}. \quad (A8)$$